Problem 53

Suppose quantity s is a length and quantity t is a time. Suppose the quantities v and a are defined by $v = ds/dt$ and $a = dv/dt$. (a) What is the dimension of v? (b) What is the dimension of the quantity a? What are the dimensions of (c) $\int v dt$, (d) $\int a dt$, and (e) da/dt?

Solution

Part (a)

Consider the dimensions of both sides of the equation.

$$\begin{aligned}[v] &= \left[\frac{ds}{dt}\right] \\ &= \frac{[ds]}{[dt]} \\ &= \frac{\mathrm{L}}{\mathrm{T}} \\ &= \mathrm{LT}^{-1}\end{aligned}$$

Part (b)

Consider the dimensions of both sides of the equation and use the result from part (a).

$$\begin{aligned}[a] &= \left[\frac{dv}{dt}\right] \\ &= \frac{[dv]}{[dt]} \\ &= \frac{\mathrm{LT}^{-1}}{\mathrm{T}} \\ &= \mathrm{LT}^{-2}\end{aligned}$$

Part (c)

Consider the dimensions of this expression and use the result from part (a).

$$\begin{aligned}\left[\int v dt\right] &= [v][t] \\ &= (\mathrm{LT}^{-1}) \cdot \mathrm{T} \\ &= \mathrm{L}\end{aligned}$$

Part (d)

Consider the dimensions of this expression and use the result from part (b).

$$\begin{aligned}\left[\int a dt\right] &= [a][t] \\ &= (\mathrm{LT}^{-2}) \cdot \mathrm{T} \\ &= \mathrm{LT}^{-1}\end{aligned}$$

Part (e)

Consider the dimensions of this expression and use the result from part (b).

$$\begin{aligned}\left[\frac{da}{dt}\right] &= \frac{[da]}{[dt]} \\ &= \frac{\mathrm{LT}^{-2}}{\mathrm{T}} \\ &= \mathrm{LT}^{-3}\end{aligned}$$